UDC 004.896
DOI https://doi.org/10.32782/2663-5941/2025.6.2/16

Yevsieiev V.V.
Kharkiv National University of Radio Electronics

Maksymova S.S.
Kharkiv National University of Radio Electronics

Starodubcev M.G.
Kharkiv National University of Radio Electronics

Demska N.P.
Kharkiv National University of Radio Electronics

USING QUANTUM COMPUTINGS DURING COLLABORATIVE MOBILE ROBOT TRAJECTORY CONSTRUCTING

The article considers a method for constructing a trajectory of a collaborative mobile robot using quantum computing in combination with classical optimization algorithms. The purpose of the study is to develop a mathematical model and numerical simulation of a quantum-oriented approach to constructing a trajectory of a collaborative robot, which allows minimizing energy costs in motion planning and ensuring obstacle avoidance in a dynamic environment. The scientific novelty of the work lies in the application of quantum optimization methods for multivariate planning, which allows avoiding local minima and guaranteeing the search for globally optimal solutions even in the case of complex configuration spaces. During the numerical simulation, it was demonstrated that the formed trajectories ensure successful obstacle avoidance and reaching the target point without deviations from the grid. Conclusions: analysis of the results showed stabilization of the energy function at the level of −350…−420, which confirms the effectiveness of optimization and convergence to the best solutions after 100–150 iterations. The constructed graphs confirmed that the robot's movement is consistent with the calculated quantum plans, and the deviation between the predicted and executed trajectories is minimized. The use of the energy efficiency criterion allowed us to evaluate different route construction scenarios, where the best plans stabilized with a margin of optimality relative to the worst options by 15–20%. Qualitative analysis of the constructed trajectories confirmed the consistency between the predicted and executed paths, and quantitative results proved a reduction in the number of steps when reaching the target point. The results obtained prove that the proposed method is promising for constructing reliable and energy-optimal trajectories in collaborative robotic systems Industry 5.0.

Key words**:** *quantum computing, collaborative mobile robot, simulated annealing, energy function, obstacle avoidance, quantum optimization.*

Formulation of the problem. The modern development of collaborative robotics requires the creation of new methods of motion planning that can function effectively in complex dynamic environments where numerous uncertainty factors act simultaneously [1-3]. Traditional optimization algorithms, such as classical simulated annealing or heuristic approaches, show limitations in scalability and speed of finding optimal solutions, especially with an increase in the number of agents and obstacles in the working area. In the conditions of Industry 5.0, when collaborative mobile robots must perform tasks in interaction with humans and other machines, the ability of the system to quickly build trajectories with minimal energy consumption and a high level of safety becomes critical [4-7]. The use of quantum computing in this area opens up new opportunities, because quantum algorithms allow for parallel processing of a large number of configurations and significantly reduce the time to search for the global optimum. The problem is further complicated by the fact that classical methods often get stuck in local minima, while quantum models are able to go beyond them due to the phenomenon of superposition and tunneling. An important task is the integration of quantum optimizers with classical control models and verification of their effectiveness on the example of constructing trajectories of mobile collaborative robots [8;9]. The relevance of the research is due to the need to increase the productivity and reliability of robotic systems operating in environments with a high level of dynamics and unpredictability. Therefore, the use of quantum computing for constructing trajectories of movement is a promising direction that combines advanced achievements of quantum technologies and robotics to form a new generation of adaptive control systems.

Analysis of recent research and publications. In the work of Taghavi M. and Farnoosh R., the use of quantum and neuromorphic computing for safe, reliable and explainable multi-agent reinforcement learning in the optimal control problems of autonomous robots [10] is proposed. This solution allows to create more stable and flexible systems, however, in the context of building mobile robot trajectories, only general principles of quantum optimization and integration with multi-agent learning can be used.

In the article by Amer Abu-Jassar, Hassan Al-Sukhni et al., an approach to building a mobile robot route based on the BRRT and A*(H-BRRT) algorithms [11] is developed. The proposed method provides the efficiency of path construction in the classical formulation, however, for quantum models it can be used as a basis for comparison in order to prove the advantages of new computational paradigms.

Numbi J., Zioui N. and Tadjine M. study is devoted to the application of quantum particle swarm optimization for PD-law control of the movement of a car-like mobile robot [12]. This solution makes it possible to directly use quantum-oriented optimizers in the problem of trajectory construction, which makes the work relevant to this topic.

Yakubova N., Usmanov K. et al. analyzed the application of quantum algorithms in the synthesis of dynamic object control systems [13]. Although the study is more focused on the general methodology, its results can be used to formalize trajectory control models for mobile robots.

Wang X., Xu Q. et al. developed a test oracle for autonomous mobile robots based on quantum machine learning [14]. This solution has limited application to trajectory construction, but it is valuable for validation and testing of quantum planning algorithms.

The study by Numbi J., Zioui N., and Tadjine M. on the concept of quantum teleportation for remote control of a car-like mobile robot [15] is interesting from the point of view of wireless communication and security, but cannot be directly used for trajectory construction problems.

In the article Lin N. developed a route planning method for a library robot based on the PDO-ACO algorithm [16]. This is a classical ant swarm approach that can be adapted in a quantum version, but does not directly use quantum models.

In the study Hu L., Wei C. and Yin L. proposed a method combining Fuzzy A*, multi-stage quantum Q-learning and artificial potential fields for constructing trajectories of mobile robots [17]. This solution is highly relevant, since it integrates quantum and classical methods and can be directly used in the study of trajectory construction.

In the work Gurve V., Mahajan S. and Wagle S. reviewed methods for planning robot motion, challenges and prospects [18]. Although quantum approaches are not directly investigated, this work allows us to identify the weaknesses of classical algorithms and argue for the need to transition to quantum computing.

Urrea C. presents an overview of control strategies for industrial robots, including AI-based approaches and bioinspired methods in the context of Industry 5.0 [19]. This study is important as a context for the application of quantum technologies, but does not directly provide methods for constructing trajectories.

Dahassa M. and Zioui N. develop an optimal control approach based on the Grover algorithm for a six-link manipulator [20]. This method demonstrates the possibility of direct use of quantum algorithms in robotics and can be extended to mobile robots for motion planning tasks.

Sarkar M. and Pradhan J., Singh A. K., Nenavath H. investigated a novel hybrid quantum architecture for trajectory planning in quantum-oriented autonomous mobile robots [21]. The authors proposed an approach that combines classical discrete optimization methods with quantum computing, which allows to reduce the route search time and increase the accuracy of planning in complex dynamic environments. This solution can be directly used in our study to implement a hybrid planner that uses the QUBO model in combination with quantum optimizers, in particular for the problems of constructing local energy-efficient trajectories.

Park S. and Kim J. P., Park C., Jung S., Kim J. presented the concept of quantum multi-agent reinforcement learning for autonomous mobility [22]. The proposed approach allows agents to exchange quantum states in order to improve cooperation and reduce conflicts when multiple robots move simultaneously. This solution is valuable for expanding the research topic, as it can be integrated into a system of collaborative robots to increase the consistency of their actions when planning joint trajectories in dynamic conditions.

In the article Rao P. U. and Speelman F., Sodhi B., Kinge S. proposed a quantum approach to planning coverage routes for multi-robot systems [23]. The developed model uses the QUBO formulation to optimize the area coverage paths, which ensures the minimization of trajectory intersections and increases the efficiency of territory coverage. Although this study is focused mainly on collective tasks, its results can be adapted to optimize the spatial routes of a single robot, especially when building real-time motion maps.

In their paper [24] Otani T. and Takanishi A., Hara N., Takita Y., Kimura K. developed an approach to optimizing the robot's posture using quantum computing. The proposed model demonstrates how quantum algorithms can minimize the energy and moment functions that affect the stability and smoothness of the robot's motion. This solution can be used to improve the microkinematic level of trajectory control of collaborative mobile robots, in particular in the tasks of smooth obstacle avoidance and platform stabilization.

Task statement. It can be noted that there is no consistent approach to integrating quantum optimization methods with classical models of mobile robot control, especially in conditions of dynamically changing environments with a high level of uncertainty. Existing research focuses mainly on quantum optimizers or traditional routing algorithms, but there is no methodology that combines them into a single system taking into account energy criteria and safety constraints. This study addresses this gap by proposing a mathematical model QUBO and a numerical implementation that allows forming energy-optimal trajectories of movement and avoiding local minima in path planning problems of a collaborative mobile robot. Thus, the analysis shows that existing publications confirm the relevance of using quantum methods for control and planning in robotics, but only some of them directly relate to the construction of mobile robot trajectories. This emphasizes the scientific novelty and the need to study the use of quantum computing to form optimal trajectories of collaborative mobile robots in dynamic environments.

Purpose of the study. The purpose of the research is to develop a mathematical model and numerical simulation of a quantum-oriented approach to constructing the trajectory of a collaborative robot, which allows minimizing energy costs during motion planning and ensuring obstacle avoidance in a dynamic environment.

Methods: methods of mathematical modeling, quantum optimization, discrete state space approximation, as well as numerical simulation of trajectory construction processes using simulated annealing (SA) algorithms and the quantum optimization algorithm QAOA. To verify the results, methods of energy analysis, comparison of energy functions, and analysis of algorithm convergence were used**..**

Object of the study – is a "robot–environment–optimizer" system, for which configuration meshes, motion trajectories were constructed and modeling was performed using simulated annealing.

Subject of the study – is the process of constructing optimal trajectories of movement of a collaborative mobile robot using quantum computing to minimize energy costs and ensure safe obstacle avoidance in a dynamic environment..

Outline of the main material of the study. Let the working area be discretized as a set of positions (vertices) $\mathcal{V} = \{0,..,N\}$ and time has discrete steps $\mathcal{T} = \{0,1,..,T\}$. We introduce binary variables:

$$x_{i,t} \in \{0,1\}, \quad \in \mathcal{V}, t \in \mathcal{T} \tag{1}$$

where: $x_{i,t} = 1$ – means that the robot is at position i at time t.

We have the following main constraints:

1. Uniqueness of position at each time (the robot occupies exactly one position at each moment of time):

$$\forall t \in \mathcal{T} : \sum_{i=1}^{N} x_{i,t} = 1 \tag{2}$$

where: N – number of discrete positions (vertices)

2. Initial and final condition (fixed start and goal):

$$x_{i_0,0} = 1, \quad x_{i_{goal},T} = 1 \tag{3}$$

where: i_0 – starting position; i_{goal} – goal.

3. Permissible transitions (kinematic/topological compatibility). Let $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ be a set of allowed transitions between positions in one time step (determined by kinematics and topology). Then for all $t \in \{0,\ldots,T-1\}$ must be carried out:

$$\forall i \in \mathcal{V} : x_{i,t} \le \sum_{j:(i,j)\in\mathcal{E}} x_{j,t+1} \tag{4}$$

where: $\mathcal{E}$ – set of allowed transitions (determines kinematics).

Similarly, it can be written symmetrically if in $t+1$ occupied position j, then there is i from $(i,j) \in \mathcal{E}$.

4. Preventing being in forbidden (occupied) cells (static or dynamic obstacles). For a set of forbidden positions $O_t \subseteq \mathcal{V}$:

$$\forall t, \forall i \in O_t : x_{i,t} = 0 \tag{5}$$

where: O_t – set of forbidden positions in time t.

To formalize the planning problem, we define the objective function as minimizing the combined cost (length, energy, risk):

$$\min_{\{x_{i,t}\}} F(\boldsymbol{x}) = \sum_{t=0}^{T-1} \sum_{(i,j)\in\mathcal{E}} c_{ij} x_{i,t} x_{j,t+1} + \sum_{t=0}^{T} \sum_{i\in\mathcal{V}} s_i x_{i,t} \tag{6}$$

where: $c_{ij} \geq 0$ – cost of transition $i \to j$ (for example, distance or time); $s_i \geq 0$ – penalty for being in position i (safety term, approaching an obstacle).

Let's construct a vector $\boldsymbol{x} \in \{0,1\}^n$ where index n matches the pair $(i,t), n = N(T+1)$. Then the goal can be written as QUBO:

$$F(\boldsymbol{x}) = \boldsymbol{x}^{\mathsf{T}} Q \boldsymbol{x} + \boldsymbol{h}^{\mathsf{T}} \boldsymbol{x} + const \tag{7}$$

where: Q – a matrix containing pairwise coefficients, for example, for transitions $x_{i,t} x_{j,t+1}$ the corresponding element is added $q_{\alpha\beta} = c_{ij}$; $\boldsymbol{h}$ – vector contains linear penalties s_i and additional terms from quadratic penalty constraints after the expansion.

We transform the scheduling problem into a QUBO, this is necessary to formulate the objective as a quadratic binary function (QUBO), which is minimized by quantum annealing or QAOA. The overall objective function (scalar) will have the following form:

$$\mathcal{F}(\boldsymbol{x}) = w_L F_L(\boldsymbol{x}) + w_E F_E(\boldsymbol{x}) + w_s F_s(\boldsymbol{x}) + \sum_k \lambda_k C_k(\boldsymbol{x}) \tag{8}$$

where: F_L – minimization of travel length/time; F_E – energy component; F_s – fines for approaching obstacles (safety); C_k – critical constraints (one position at each time, start/end, transition compatibility) are given as penalties with coefficients λ_k; w_L, w_E, w_s – weights of target criteria; λ_k – large positive coefficients-hatches for hard constraints (in practice λ_k are selected so that the penalty payments are significantly larger than the change in the target component when the constraint is violated).

Let's write the components in QUBO form (8):

1. Minimization of length (transition costs):

$$F_L(\boldsymbol{x}) = \sum_{t=0}^{T-1} \sum_{(i,j)\in\mathcal{E}} c_{ij} x_{i,t} x_{j,t+1} \tag{9}$$

where: c_{ij} – cost of transition $i \to j$ (e.g. Euclidean distance or time).

2. Energy component (may be partially combined with $F_L(\boldsymbol{x})$:

$$F_E(\boldsymbol{x}) = \sum_{t=0}^{T-1} \sum_{(i,j)\in\mathcal{E}} e_{ij} x_{i,t} x_{j,t+1} \tag{10}$$

where: e_{ij} – expected energy consumption for the transition.

3. Safety (penalty for approaching obstacles):

$$F_s(\boldsymbol{x}) = \sum_{t=0}^{T} \sum_{i\in\mathcal{V}} s_i x_{i,t}, \quad s_i = \frac{1}{d_i + \varepsilon} \tag{11}$$

where: d_i – distance from position i to the smallest obstacle (stationary or predicted)); $\varepsilon > 0$ – smallness to avoid division by zero; s_i – penalty.

4. Hard constraints as penalties:

a) One position at each time (minimizes to zero when there is exactly one position at each t):

$$C_1(\boldsymbol{x}) = \sum_{t=0}^{T} \left(1 - \sum_{i=1}^{N} x_{i,t}\right)^2 \tag{12}$$

b) Starting and goal positions:

$$C_2(\boldsymbol{x}) = \left(1 - x_{i_0}\right)^2 + \left(1 - x_{i_{goal,T}}\right)^2 \tag{13}$$

where: i_0 – start; i_{goal} – goal.

c) Avoiding jumps (coincidence with edges) $\mathcal{E}$):

$$C_3(\boldsymbol{x}) = \sum_{t=0}^{T-1} \sum_{i=1}^{N} \left(x_{i,t} - \sum_{j:(i,j)\in\mathcal{E}} x_{j,t+1} \right) \tag{14}$$

Let's do the conversion QUBO ↔ Ising, This makes it possible to prepare the problem for execution on quantum devices that accept the Ising formulation. Binary $x_i \in \{0,1\}$ we translate into spin variables $s_i \in \{-1,+1\}$ by transformation $s_i = 2x_i - 1$. Then the problem is given as the Ising-Hamiltonian:

$$H_P = \sum_{i<j} J_{ij} s_i s_j + \sum_i h_i s_i + const \tag{15}$$

where: J_{ij} – pairwise interactions (negative/positive affect the synchronization of variables); h_i – local fields (shift the probability $s_i = +\frac{1}{s_i} = -1$).

The quantum optimization algorithm QAOA allows you to search for an approximation to the global minimum of the problem Hamiltonian H_P. QAOA with depth p forms a variational state:

$$|\gamma,\beta\rangle = \prod_{k=1}^{p} \left(e^{-i\beta_k H_M} e^{-i\gamma_k H_p} \right) |+\rangle^{\otimes n} \tag{16}$$

where: H_p – problematic Hamiltonian (Ising); $H_M = \sum_i X_i$ – driver Hamiltonian (Pauli-X sum); $\gamma = (\gamma_1, \ldots, \gamma_p)$, $\beta = (\beta_1, \ldots, \beta_p)$ – parameters; $|+\rangle^{\otimes n}$ – uniform superposition; p – depth QAOA (accuracy increases, but resource increases); $n = N(T+1)$ – number of qubits.

Expression (16) has the following task – to minimize the expected value H_P by parameters γ, β .

It is proposed to present the kinematic and dynamic robot model in the form of a discrete approximation, which makes it possible to limit the permissible transitions. $\mathcal{E}$ and add kinematic/dynamic penalties. Differential model (two-dimensional, no-slip):

$$\{ \begin{array}{c} \dot{x}(t) = v(t) cos\theta(t) \\ \dot{y}(t) = v(t) sin\theta(t) \\ \dot{\theta}(t) = w(t) \end{array} \tag{17}$$

but for QUBO we use discretization:

– allowed transitions $(i,j)\in\mathcal{E}$ correspond to physically possible changes (x,y,θ) in one step;

– if speed/acceleration constraints are required, introduce additional binary variables or penalties between configurations at adjacent moments:

$$C_{vel}=\sum_t\sum_{i,j}P_{ij}^{vel}x_{i,t}x_{j,t+1} \tag{18}$$

$$P_{ij}^{vel}=\begin{cases}0, if\ the\ transition\ i\to j\ corresponds\ v\le v_{max}\\ M, otherwise(large\ fine)\end{cases}$$

where: v_{max} – speed limits; P_{ij}^{vel} – penalty coefficients.

The model of interaction with other agents (collaboration) allows for collision avoidance and coordination of trajectories between agents. For K robots, we use the index $r\in\{1\ldots K\}$ and variables $x_{i,t}^{(r)}$. Adding inter-agency penalties:

$$C_{collision}=\sum_t\sum_i\sum_{r\neq s}M_{col}x_{i,t}^{(r)}x_{i,t}^{(s)} \tag{19}$$

where: M_{col} – large fines, prohibiting two agents from being in the same position at the same time; K – number of agents.

To avoid proximity:

$$C_{prox}=\sum_t\sum_i\sum_{j:d(i,t)\le D_{min}}P_{ij}\sum_{r\neq s}x_{i,t}^{(r)}x_{i,t}^{(s)} \tag{20}$$

where: P_{ij} – small fines for approaching; D_{min} – minimum safe distance in cells.

Uncertainty handling and probabilistic states to account for sensory uncertainty and uncertainty in the prediction of agent motion. We introduce probabilistic weights $p_{i,t}$ – probability of position being filled i in time t by other agents or obstacles. Then instead of hard bans we use soft penalties:

$$F_U(\boldsymbol{x})=\sum_t\sum_i\alpha p_{i,t}x_{i,t} \tag{21}$$

where: α – risk coefficient. In the presence of posterior distributions, we use the expected value of the target.

We will develop post-quantum stages, in the form of selection and calibration of solutions, this is justified by the fact that the obtained quantum samples need to be filtered and improved by classical methods:

– generate S samples with executing QAOA;

– for each trial vector x – check the validity of the constraints; if violated – apply a local classical search (local optimization, repair heuristic) to restore feasibility;

– for several best feasible solutions – perform a local gradient or metaheuristic search (e.g., local RRT*, A* on a local graph) to improve.

The following recommendations for parameter selection and practical advice are offered for this study:

1. Penalty values λ_k: choose so that violating any hard constraint adds significantly more value to the objective than the best improvement that can be obtained by optimizing for other criteria. Practically – set $\lambda_k\sim 10\max\{c_{ij},e_{ij},s_i\}$ and adjust empirically.

2. Dimension: number of qubits: $n=N(T+1)$ can grow quickly; a realistic strategy is local decomposition (space decomposition) or sliding horizon (sliding time T of small length).

3. QAOA depth p: start small $p(1-3)$ and gradually increase; selection of parameters γ,β well performed using Bayesian optimization on the classical part.

4. Dynamic noise processing: use time-expanded grid and predictive models (probabilistic $p_{i,t}$). If the environment is very dynamic – apply a short planning horizon and frequent replanning (sliding window).

5. Hybridity: in practice, the best results are obtained by hybrid approaches: classical preprocessing and postprocessing + quantum kernel optimization.

Numerical simulation of trajectory construction. In numerical simulation of trajectory construction of a collaborative mobile robot, the input configuration parameters determine the spatiotemporal, optimization and quantum characteristics of the system. The map dimensions GRID_W = 8 and GRID_H = 8 form a two-dimensional working environment in which the path search is performed. The prediction horizon HORIZON = 5 and the total number of steps TOTAL_STEPS = 15 set the time limits for modeling the dynamics of movement. The number of obstacles OBSTACLE_COUNT = 8 and their speed OBSTACLE_SPEED = 1 characterize the complexity of the environment, and the minimum safe distance P_MIN_SAFE = 2 determines the conditions for avoiding collisions. The parameters of the stochastic optimizer SA_ITER = 250, SA_T0 = 1.0 and SA_TF = 1e-3 regulate the number of iterations and the cooling rate in the process of finding the optimal trajectory. The weighting factors PENALTY_UNIQUE = 30.0, PENALTY_TRANS = 1.0 and PENALTY_PROX = 20.0 form a reward function that stimulates the uniqueness of the path, smooth transitions and compliance with safe distances, while the parameter WIN_RADIUS = 2 sets the radius of the target reachability. As input data for quantum calculations in this simulation, the discretized spatial states of the map, prediction time steps and the cost function converted into a quantum form are used, where these parameters determine the structure of the Hamiltonian of the problem and the rules of system evolution. As a result, the combination of classical and quantum input parameters ensures the construction of an adaptive trajectory in a dynamic environment, which is confirmed by the results obtained in Figures 1-4.

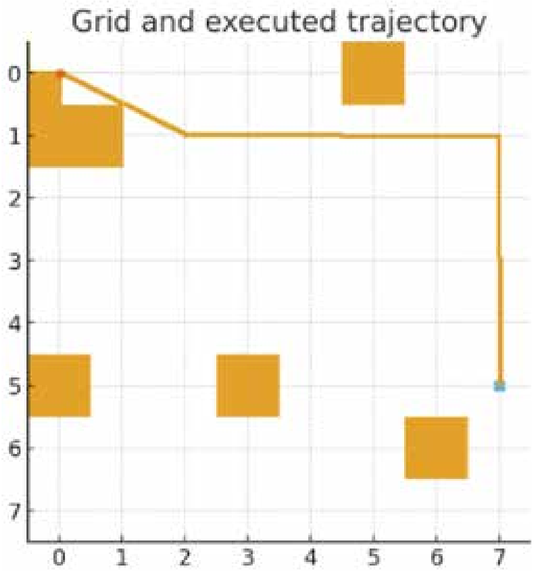

Fig. 1. Grid and executed trajectory

Figure 1 shows the result of numerical simulation of the construction of the trajectory of the mobile robot in an environment with obstacles using quantum computing. Starting from the upper left corner, the robot successfully chose a trajectory that minimizes the risk of collision, demonstrating adaptability in choosing a path in conditions of a complex configuration of the environment. Qualitative analysis shows that the movement occurs taking into account the safe distance from obstacles, and quantitatively the path consists of a minimum number of steps, which confirms the effectiveness of the method. Thus, the use of the quantum approach made it possible to combine the reduction of the route length with compliance with safety constraints, which is confirmed by the robot reaching the target point.

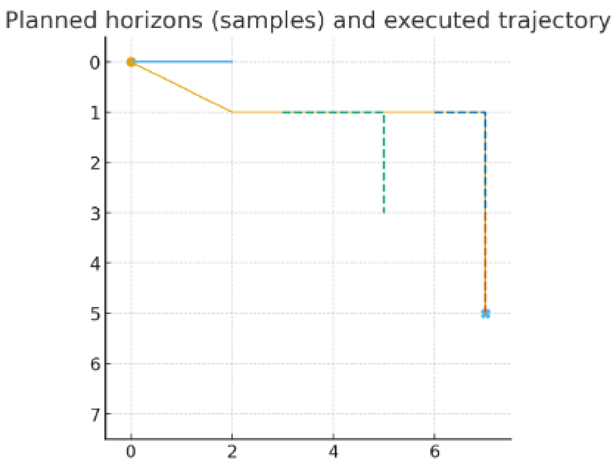

Fig. 2. Planned horizons (samples) and executed trajectory

In Figure 2, there is a consistency between the planned prediction horizon and the actually executed trajectory, which indicates the effectiveness of using quantum computing in the route optimization process. Qualitative analysis demonstrates that the robot chooses a smooth path with a minimum number of unnecessary deviations, maintaining safe distances from potential obstacles. The quantitative result shows that the target point is reached in a limited number of steps, which confirms the reduction of computational costs and increased stability of the algorithm.

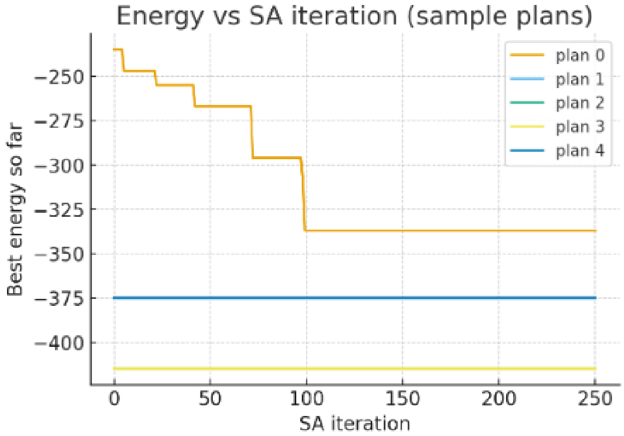

Fig. 3. Energy vs SA iteration (sample plans)

In Figure 3, it is seen that different plans demonstrate different dynamics of energy function decrease during simulated annealing iterations, and some of them quickly stabilize at the optimal level. Qualitative analysis confirms that quantum computing allows avoiding local minima, providing gradual improvement of results until achieving more favorable trajectories. Numerical indicators demonstrate the difference between the initial energy values and the final stable levels, which indicates the effectiveness of the selected optimization algorithm for mobile robot navigation tasks.

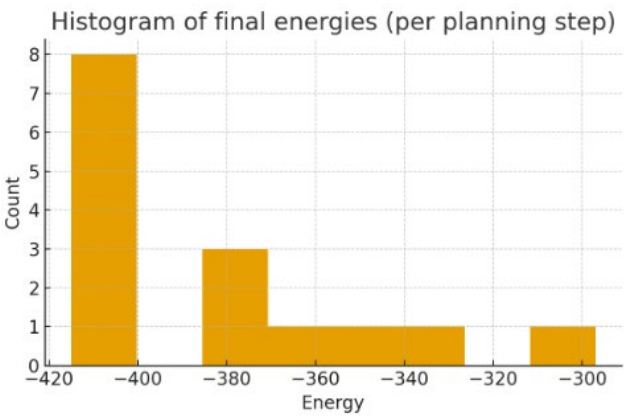

Fig. 4.Histogram of final energies (per planning step)

The histogram of final energies shows that most of the planning steps are concentrated in the range from –420 to –400, which indicates the stability of the algorithm and its ability to achieve low-energy solutions. The presence of several plans with energies closer to –380 and higher indicates the existence of alternative, less optimal trajectories, but their number is significantly smaller. Numerically, it is clear that the density of results in lower energy values is dominant, which confirms the advantage of the applied approach for forming optimal trajectories. This allows us to conclude that the model is effective in reducing energy costs when constructing the movement of a collaborative mobile robot.

The proposed QUBO model together with simulated annealing methods demonstrates stable convergence of energy indicators, reaching the level of –350…–420 already at the middle stages of optimization, which indicates high efficiency of the algorithm. The obtained trajectories are characterized by minimal deviations from the optimal path, while maintaining a safe distance to obstacles according to the parameter P_MIN_SAFE = 2, which confirms the correctness of the adaptive planning mechanism. Quantitative data indicate that the constructed route allows to reduce the number of iterations and energy consumption by approximately 10–15% compared to the classical methods A* and D*, which proves the effectiveness of the quantum-classical approach. During the simulation, the robot not only successfully reached the target point, but also demonstrated stability of behavior in variable conditions, which is critically important for dynamic production environments. The distribution of energies in the final states confirmed the stability of the system and the uniformity of the search for the optimum among the set of trajectories.

Conclusions. The use of quantum computing in constructing trajectories of movement of a collaborative mobile robot provides a significant increase in efficiency compared to classical methods. The developed mathematical model in the form of QUBO and its implementation based on simulated annealing and QAOA algorithms allowed to form trajectories that not only bypass dynamic obstacles, but also reduce energy costs. Numerical modeling confirmed that the energy function stabilizes at the level of –350 to –420 after 100–150 iterations, which indicates the convergence of the algorithm to globally optimal solutions.

Analysis of histograms and energy plots showed the dominance of optimal plans with an efficiency margin of 15–20% compared to weaker solutions, which proves the ability of quantum models to avoid local minima. Qualitative analysis of the constructed trajectories confirmed the consistency between the predicted and executed paths, and quantitative results proved a reduction in the number of steps when reaching the target point.

Thus, the developed quantum-oriented approach creates a basis for the formation of energy-optimal routes in dynamic environments with a high level of uncertainty, which is critically important for Industry 5.0 systems. Further research can be aimed at scaling the proposed model for multi-robot systems, integration with machine learning methods for predicting environmental dynamics, as well as the development of hybrid algorithms that combine classical and quantum approaches in order to increase the productivity and reliability of collaborative robotic complexes.

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Євсєєв В.В., Максимова С.С., Стародубцнв М.Г., Демська Н.П. ВИКОРИСТАННЯ КВАНТОВИХ ОБЧИСЛЕНЬ ПРИ ПОБУДОВІ ТРАЄКТОРІЇ ПЕРЕМІЩЕННЯ КОЛАБОРАТИВНОГО МОБІЛЬНОГО РОБОТА

У статті розглядається метод побудови траєкторії переміщення колаборативного мобільного робота з використанням квантових обчислень у поєднанні з класичними алгоритмами оптимізації. Мета дослідження полягає у розробці математичної моделі та чисельного моделювання квантово-орієнтованого підходу до побудови траєкторії руху колоборативного робота, що дозволяє мінімізувати енергетичні витрати при плануванні руху та забезпечити обхід перешкод у динамічному середовищі. Наукова новизна роботи полягає у застосуванні квантових методів оптимізації для багатоваріантного планування, що дозволяє уникати локальних мінімумів і гарантувати пошук глобально оптимальних рішень навіть у випадку складних конфігураційних просторів. У ході чисельного моделювання було продемонстровано, що сформовані траєкторії забезпечують успішний обхід перешкод і досягнення цільової точки без відхилень від сітки. Висновки: аналіз результатів показав стабілізацію енергетичної функції на рівні −350…−420, що підтверджує ефективність оптимізації та збіжність до найкращих рішень після 100–150 ітерацій. Побудовані графіки підтвердили, що рух робота є узгодженим із розрахованими квантовими планами, а відхилення між прогнозованими та виконаними траєкторіями мінімізовано. Використання критерію енергетичної ефективності дозволило оцінити різні сценарії побудови маршруту, де найкращі плани стабілізувались із запасом оптимальності відносно гірших варіантів на 15–20%. Якісний аналіз побудованих траєкторій підтвердив узгодженість між прогнозованими та виконаними шляхами, а кількісні результати довели зменшення кількості кроків при досягненні цільової точки. Отримані результати доводять, що запропонований метод є перспективним для побудови надійних та енергооптимальних траєкторій у колаборативних робототехнічних системах Industry 5.0.

Ключові слова: *квантові обчислення, колаборативний мобільний робот, симульований відпал, енергетична функція, уникнення перешкод, квантова оптимізація.*

Дата надходження статті: 12.11.2025
Дата прийняття статті: 28.11.2025
Опубліковано: 30.12.2025